Combinatorics 2

15 July 2023 11:35

· SET THEORY

Definition of Set :- A set is a collection of well-defined distinct objects

A = { emotions of human being } → ~~well~~ not well defined so not set

$A = \{1, 2, 3, \dots, n\}$ → The first n Natural numbers
→ It is a set

$A = \{1, 2, 2, 3\}$ → so not set
repetitions

$A = \{1, 2, 4, 8, \dots\}$ → powers of 2
→ It is set

$A = \{|z| = 1\} \cup \{|z| = 2\}$
→ both well define
⇒ It is a set

$A \cup B = \{a \in (A \text{ or } B)\}$

$A \cap B = \{a \in (A \text{ and } B \text{ both})\}$

$A = \{ |z| = 0 \text{ and } z \in \mathbb{R}^+ \}$ = empty set = ϕ
empty is a set with no objects

$a \in A$
a is a element

$a \subset A$
a is a subset
a is itself a set also

A is a set, B is a set

$A = B \Rightarrow$ for every $a \in A \ \exists \ a \in B$ and vice-versa

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$A = \{ |z| = 3n \ ; n \in \{0, 1, 2\}, \ Re(z) = 5 \}$

$A = \{ 5 + \sqrt{11}\, i, \ 5 - \sqrt{11}\, i \}$

$|A| = 2$ → Cardinality Symbol

$|z| = 0$

$|5 + yi| \neq 0$

$|5 + yi| \neq 3$

$|5 + yi| = 6$

$5^2 + y^2 = 6^2$

$y^2 = (6^2 - 5^2)$

$y = \pm\sqrt{6^2 - 5^2} = \pm\sqrt{11}$

$A \cap B = \phi$ → empty set
⇒ $\forall \ a \in A \ \nexists \ a \in B$

U... ...l Set :- generally notation U (it may be already defined or defined by us)

Universal Set :- generally notation U (it may be already defined or defined by us)

$A \cap A^c = \phi$

$A^c \cap B$ → to find out we must know U

Example :-

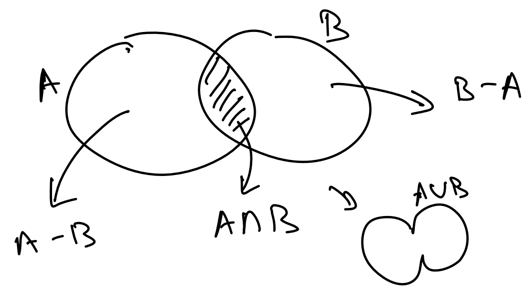
$A = \{1, 2, 3\}$ $\quad B = \{1, 2, 4, 8, 16 \ldots\}$

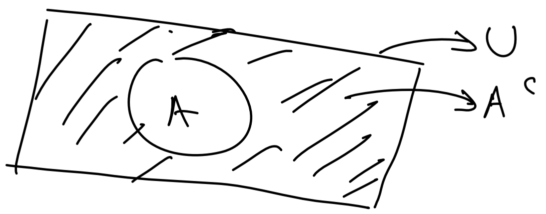
$A^c \cap B =$ let U be $\mathbb{N}$ then $A^c = (\mathbb{N} - \{1, 2, 3\})$

$A^c \cap B = \{4, 8, 16 \ldots\}$

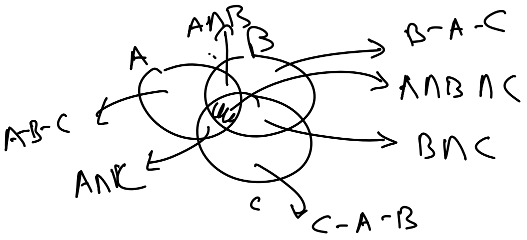
Venn Diagram :- A, B sets.

$\underline{A - B} = \{ \underline{a \in A \text{ and } a \notin B} \} = (A \setminus (A \cap B))$





A, B, C sets



$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |C \cap B| + |A \cap B \cap C|$

$|(A \cup B) \cup C|$

$= |A \cup B| + |C| - |(A \cup B) \cap C|$

$= (|A| + |B| - |A \cap B|) + |C| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$$\boxed{|A_1 \cup A_2 \cup A_3 \ldots \cup A_n| = \sum |A_i| - \sum |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k| - \ldots (-1)^n (A_1 \cap A_2 \cap \ldots \cap A_n)}$$

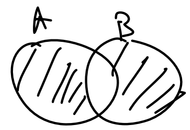
$\bigcup A_i = A_1 \cup A_2 \cup \ldots A_n$

$\bigcap A_i = A_1 \cap A_2 \cap \ldots A_n$

Symmetric Difference :-

A, B set

$$A \Delta B = (A-B)\cup(B-A)$$
$$= A\cup B - A\cap B$$



$$(A_1\cup A_2)\cup A_3 = A_1\cup(A_2\cup A_3) \qquad (A_1\cap A_2)\cap A_3 = A_1\cap(A_2\cap A_3)$$

$$A_1\cup A_2 = A_2\cup A_1 \qquad A_1\cap A_2 = A_2\cap A_3$$

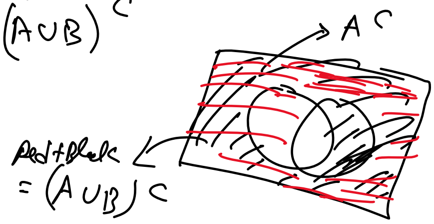
De Morgan's Law :-

A, B sets

HW to check using algebra of set theory

(i) $(A\cup B)^c = (A^c\cap B^c)$

(ii) $(A\cap B)^c = (A^c\cup B^c)$



Prove that $A\cup B = A\cap B$ is false. → By counter-example